Chapter in *Principles of Effective On-line Teaching*

Mathematics On-line: A Virtual Reality or Impossibility

Robert Mayes
Jennifer Luebeck
Michael Mays
Erin Niemiec

Introduction

Mathematics presents many unique challenges for on-line teaching. The most noticeable barrier to communication is that mathematics is a subject heavily laden with symbolism, challenging students and instructors to express and exchange mathematical concepts electronically. Furthermore, mathematical representations often involve iconic or pictorial representations that are difficult to create and replicate on-line. Equally daunting are the abstract and algorithmic nature of mathematics, qualities that are often difficult to express in standard verbal language. Communicating mathematical ideas over the Internet is posited by some as being impossible.

Exploring content and pedagogy from a mathematics education perspective also presents challenges. Best practices in mathematics instruction call for demonstrating multiple ways of presenting a concept, promoting mathematical discourse, and acquiring skill with various forms of technology. Given these perceived barriers, why should one even try to teach mathematics courses at a distance? The first response is related to need: there is an ever growing demand for mathematics and mathematics education courses for place-bound students and teachers who are unable to leave their classrooms. The second response is related to opportunity: emerging technologies are providing novel solutions to the difficulties inherent in teaching mathematics via distance delivery models.

Distance education takes on multiple instantiations and is constantly changing. Bernard and Abrami (2004) identify five generations of distance education: print-based correspondence; broadcast TV, radio, and tapes; teleconferencing and hypertext; computer-mediated Internet; and on-line interactive media, including Internet-based access to the World Wide Web. Bennett and Bennett (2002) give some of the many pedagogical advantages Internet-based courses have over other platforms. They note that Internet courses tend to be interactive so students have the opportunity to be more engaged active learners; that the nature of communication in the courses gives a student greater access to faculty members; and that the structure of courses fosters communication among students through the development of small learning communities. These benefits have not gone unnoticed in the higher education community. In a 2003 study, Tabs found that credit-bearing Internet courses were at that time offered by more than 55 percent of all 2-year and 4-year universities. Further, 88 percent of institutions surveyed in that study planned to start using or to increase the number of distance education courses they offered, emphasizing asynchronous computer-mediated Internet instruction as a primary mode of instructional delivery (Tabs, 2003).

This chapter will describe our experiences with these fourth and fifth generations of distance education—computer mediated approaches, both synchronous and asynchronous, that allow for two-way communication and take advantage of Web-based tools and resources. We begin with a review of what current research and experience proclaim to be the state of the art in teaching via distance. Within this context, we look more closely at themes and issues of on-line learning that are particularly relevant in mathematics. We then describe the on-line programs at our respective universities (West Virginia University and Montana State University) that currently serve graduate students, many of them practicing teachers, involved in mathematics education. Finally, we conclude the chapter by sharing insights and provide suggestions gleaned from the experiences of students and instructors in our programs.

Review of Research: Distance Education

The findings discussed here are based on a grant-supported review of the literature conducted in 2004 and updated in 2006. We have restricted our review to post-1990 articles, and have made a special effort to identify articles that address the teaching of mathematics and mathematics education courses via distance. The literature reveals that efforts to provide distance delivery of mathematic courses encounter many of the same barriers that have been identified in the literature for other content areas. In our ensuing discussion, we will focus on how these barriers impact mathematics in a unique way.

One salient point about the distance education literature is the lack of evidence of quality research in this area. Moore (2001) performed a detailed analysis of distance education research over a 10-year period, finding that 74.8% of articles or dissertations classified as research conducted only descriptive research, with only 6% implementing an experimental design. He found the top three categories addressed in the research to be design issues, learner characteristics, and strategies to increase interactivity and active learning. None of these address specific issues of teaching in a content area. Bernard et al. (2004) confirm a prevailing view that distance education research is of low quality. So while we can frame our discussion on findings from the literature, this will not be sufficient. To discuss the teaching of mathematics via distance, we will also draw on the extensive experience of our respective universities in offering computer-mediated distance courses for graduate students in mathematics and mathematics education.

As distance education has blossomed in the past two decades, a substantial number of reviews (Berge & Mrozowski, 2001; Jung & Rha, 2000, Merisotis & Phipps, 1999; Moore & Thompson, 1990; Saba, 2000; Schlosser & Anderson, 1994) and meta-analyses (Allen, et al., 2002; Bernard, et al., 2004; Cavanaugh, 2001; Machtmes & Asher, 2000; Shachar & Neumann, 2003; Ungerleider & Burns, 2003) have been conducted in this area. Some of these reviews attempt to compare distance education with classroom instruction; many of them have produced recommendations about best practices in distance education. We focused on two seminal reviews, using their general findings regarding distance education to generate a set of characteristics that significantly impact the teaching and learning of mathematics on-line.

The first of these two documents, "What's the Difference?" produced by Merisotis and Phipps in 1999, was one of two studies commissioned by the Institute for Higher Education Policy related to distance education. This document provides a narrative review of the current state of research on distance education up to 1999. The second document, "How does Distance Education Compare with Classroom Instruction?" (2004) reports on a meta-analysis of distance education research conducted by Bernard et al.

Merisotis and Phipps (1999) reviewed research, expository articles, and policy papers from 1990 through 1999 to provide a basis in theory for distance education policy. They found that the vast majority of articles on distance education were opinion pieces, how-to-articles, and second-hand reports with no quality research basis. Merisotis and Phipps identified approximately 40 articles that were classified as original research, including experimental, descriptive, correlational, and case studies, and classified them based on three broad measures of the effectiveness of distance education:

- Student outcomes, such as grades and test scores;
- Student attitudes about learning via distance education; and
- Overall student satisfaction toward distance learning.

The majority of the original research articles indicated that distance education had positive outcomes in all three of these areas. The experimental studies concluded that distance learning courses compare favorably with classroom-based instruction, with students receiving similar grades or test scores and having similar attitudes towards the course. The descriptive analysis and case studies concluded that students and faculty have a positive attitude toward distance learning. Merisotis and Phipps found significant problems with the quality of the research conducted, however, and advised that the lack of quality renders many of the findings inconclusive.

In titling their report “What’s the Difference?” Merisotis and Phipps acknowledge their dispute with the conclusions drawn by Russell in his paper, “No Significant Difference Phenomenon” (1999). Russell’s compilation of more than 355 sources dating back to 1928 suggested that the learning outcomes of students in distance education courses are similar to those of students participating in traditional classrooms, implying that distance education is as good as that in traditional classrooms. Overall Merisotis and Phipps questioned the effectiveness of distance education, countering Russell’s claim of no significant difference.

Merisotis and Phipps conclude their report by suggesting that improving distance education is a question not of technology, but of pedagogy – the art of teaching. Perhaps this finding is not surprising, considering the report was commissioned by the American Federation of Teachers and the National Education Association. They challenge educators to re-examine the Seven Principles for Good Practice in Undergraduate Education promulgated by the American Association for Higher Education (Chickering & Gamson, 1987) as a focus for distance education. The AAHE’s principles of good practice promote teaching methods that:

- Encourage contacts between students and faculty
- Develop reciprocity and cooperation among students
- Use active learning techniques
- Give prompt feedback
- Emphasize time on task
- Communicate high expectations
- Respect diverse talents and ways of learning

Bernard et al. (2004) question any narrative review of distance education literature, stating their concerns about potential subjectivity and bias in selection of articles included in the review and an inability to answer questions about the magnitudes of effects. Bernard and Abrami (2004) conducted their own meta-analysis comparing distance education with classroom instruction. They categorized recent distance education research into six categories, of which three—media, demographics (including subject area), and pedagogy—directly relate to the teaching of mathematics and mathematics education via the Internet. They then listed nine significant aspects of pedagogy identified in the research literature, including face-to-face contact, mediated communication, student-teacher and student-student contact, and problem-based learning.

Bernard et al. (2004) found that achievement in distance education is widely varied and that favorable comparison with classroom instruction depends heavily on how distance education is implemented. They found asynchronous instruction to be superior to synchronous interaction, but it should be pointed out that they define “synchronous” instruction specifically as video-conferencing, omitting other possibilities for interacting in “real time” through chat rooms and other media. Overall, their meta-analysis revealed that methodology and pedagogy are more important then media in predicting success; however, the academic subjects of mathematics, science and, engineering appeared to be best suited for face-to-face instruction rather than distance education. Finally, they noted a small but significant difference in overall attitude outcomes that favored classroom instruction over distance education; similar effects were found for retention.

In their discussion, Bernard et al. (2004) acknowledge the potential power of distance education to implement constructivist based approaches, provide effective interpersonal interaction, and create learner-centered environments. They call for distance education to incorporate cognitive tools that encourage authentic learning experiences and interactivity among students. They claim that communication, both face-to-face and computer-mediated, is essential to a successful distance course and recommend that computer-mediated instruction in particular incorporate active collaborative learning experiences, which seemingly result in both better achievement and better attitude outcomes. Significantly, the researchers state that distance education should not simply replicate the traditional classroom. They express concerns about the paradoxical effects of ever more powerful computer-mediated tools that emulate face-to-face instruction, warning that this may actually lead instructors to return to lecture-based, teacher-centered pedagogies.

In addition to drawing from the two major reports cited above, we have conducted our own extensive review of the literature seeking distance education articles, particularly those with a focus on mathematics and mathematics education. The articles we located are evenly divided between expository (22) and research (21) articles. The research articles were classified as quantitative (14) and qualitative (8), with one having elements of both methodologies. Emphasis in the body of literature was heavily weighted towards affective issues (29) versus cognitive issues (6). The levels of instruction studied include high school (6), community college (7), college (22), and graduate coursework (5). The coursework itself included mathematics (15), education (7), English (2), computing (2), physics, statistics, medical care, and five articles with multiple subject areas combined. The delivery platforms used were even more varied, with the majority using a computer-mediated, Internet-based mode (21). Many courses used multiple modes, including telecourse or two-way video (6), CD or video tape (5), whiteboard (3), telephone bridge (2), synchronous audio (2), computer algebra system (2), on-site facilitator (2), textbook-centered (2), fiber optics, programming, or a tutorial system.

The research articles provide insights through literature reviews, findings, and conclusions, while the expository articles provide experience-based recommendations. In several cases, the authors have produced lists of key issues and recommendations relevant to distance learning. For example, after studying six U.S. institutions identified as having exemplary distance education programs, Carnevale (2000) produced “Quality on the Line: Benchmarks for Success in Internet-based Distance Education,” which reported twenty-four benchmarks for quality distance education. Taylor and Mohr (2001) provide another extensive list of principles to guide distance education.

In Table 1, we summarize what we consider to be key characteristics of quality distance education drawn from both the narrative and meta-analysis reviews cited earlier as well as our independent literature review spanning 2000-2006. We have organized these characteristics into several broad themes that significantly influence the effectiveness of distance education. In creating Table 1 and in the ensuing discussion of each entry, we have limited our interpretation of "distance education" to include only Internet-based forms of distance delivery and have focused our attention on characteristics we believe to have the most influence on teaching mathematics and mathematics education courses.

Table 1
Themes for Quality Distance Education

Themes	Indicators
Audience *What do we know about our learners?*	• Learner characteristics o Demographics o Diversity (e.g., learning styles) o Technical fluency o Motivation o Anonymity
Medium *What cognitive and technological tools do we use to support learning?*	• Influence of technology-based factors o Transparent vs. transformative use of technology o Embedded cognitive tools (e.g., whiteboards) o Access to print and electronic resources o Synchronous vs. asynchronous delivery
Community and Discourse *What social and interactive tools do we use to support learning?*	• Role of community in effective on-line learning • Building community through course structure • Building community through discourse • Building community through face-to-face meetings
Pedagogy *What instructional strategies do we use to support learning?*	• Active, problem-based, learner-centered instruction • Varied levels of interaction (e.g., learner-instructor vs. learner-learner) • Multiple modes of interaction (e.g., one-on-one vs. group) • Workload and awareness of student needs
Assessment *How do we assess on-line learning?*	• Prompt formative feedback • Quality, honesty, and security in assessment • Effective measures of student understanding
Content *What unique challenges arise in presenting content?*	• Symbolic nature of mathematics • Iconic/visual nature of mathematics • Abstract nature of mathematics • Mathematical technology (e.g., software and calculators)

Themes from the Literature: On-line Instruction

Audience

Demographics. Perez and Foshay (2002) found that students enrolled in distance education are predominantly older, non-traditional students and females. They noted that females may find the self-discipline and self-pacing required in distance learning to be negative aspects, lacking the social interaction and teamwork of on-site learning. Despite this, distance education offers a viable alternative to those who are placebound by family or location. Ryan (1996) noted that students from small rural communities were able, via distance education, to take courses in mathematics not offered in their schools. This allowed them to overcome a traditional disadvantage they had in starting college calculus as postsecondary students.

Diversity. Just as in the face-to-face classroom, an instructor must be aware of student diversity, although this becomes more challenging in a distance education setting. In addition, some student characteristics can become exacerbated in an on-line environment. For example, a student's learning style (e.g., active vs. reflective, extrinsic vs. intrinsic, visual vs. verbal) may have an extensive impact on his or her ability to be successful in a distance course. Diversity is also evident in differences in gender, ethnic background, professional experience, and location. The diversity inherent in an on-line student population can be exploited to add depth and perspective to learning experiences (Luebeck & Bice, 2006).

Technical fluency. Merisotis and Phipps (1999) acknowledged the need for special skills on the part of students and instructors in an on-line environment. Huff (2002) identified technical skills in the areas of basic computing, computer communications, and computer applications as critical for students in order to successfully complete an on-line course**.**

Motivation. Student attitude, motivation, and skill level all influence time spent studying and attitude about workload. Parker (2003) found that a student's locus of control, or level of self-motivation, is correlated with academic persistence at a ratio of 0.83 ($p=.05$). Parker (2003) suggests that the self-motivated student is most likely to complete a course via distance. An extrinsic, active learner may find the support provided in an on-line course to be inadequate. On the other hand, an intrinsic, reflective learner may thrive in the self-directed atmosphere provided by an on-line environment.

Anonymity. Students are drawn to distance education courses not only for their convenience and flexibility (Sullivan, 2001), but also for the measure of anonymity they provide. The divested authority inherent in distance education encourages students to challenge the instructor. The anonymity of distance education provides a positive learning environment for shy or quiet students, math-anxious students, and females, who prefer a less masculine style of discourse that is not dominated by face-to-face, in-class confrontations (Smith, Ferguson, & Caris, 2001; Sullivan, 2001; Taylor & Mohr, 2001).

Medium

Transformative technology. The efficacy of computer-mediated learning is related to whether we view the technological medium as transparent or transformative (Bernard et al., 2004). The transparent view supports the "no significant difference" phenomenon between classroom- and distance-based instruction. In this view, the technological medium is not seen as the most important factor affecting student learning and satisfaction–more important are subjective factors such as learner characteristics (e.g., motivation and learning style) and the quality of instruction and learning tasks. In essence, the transparent view implies that on-line instruction is a relatively neutral means of delivering a course, with effectiveness determined by the same influences that exist in a face-to-face format.

The transformative view values the innovative strategies and variety of resources that can be brought to bear in an on-line course, implying that these constitute an advantage for distance education over the face-to-face classroom. Transformational adherents hypothesize that distance education media may result in increased reflection due to writing and peer interaction on-line (Hawkes, 2001), development of writing skills (Winkelmann, 1995), and improved problem solving and critical thinking due to peer modeling and mentoring (Garrison, Anderson, & Archer, 2001; Lou, 2004; Lou, Dedic, & Rosenfield, 2003; Lou & MacGregor, 2002; McKnight, 2001).

Cognitive tools. Ongoing technological advancements in communication and cognitive tools have helped to create an active on-line learning environment. Two-way audio and video communication can be broadcast over the Internet to enhance text-based instruction. Electronic whiteboards for written communication, formative assessment tools that allow active student engagement, Java-based interactive applets, and the ability to share software contribute to a more authentic learning experience. Computer-mediated course software now emulates the live classroom, breaking down the barriers that separate on-line and classroom-based instruction. However, these new media do have a down side. As the interface with students more closely emulates the live classroom, there is less demand on instructors to depart from their innate lecture-based, teacher-centered traditional approaches. In effect, the advancement of the medium could undermine the transformative process (Bernard et al., 2004).

Access to resources. Along with the messages, materials, and lectures posted within an on-line course, students have access to a wealth of external resources. Electronic reserve capabilities at many university libraries have replaced textbooks with a set of easily accessed, relevant readings. Students may conduct research projects using virtual libraries, experiment with applets and other interactive tools, or dialogue in "real time" with a guest expert in the field using a chat room. Web links can be embedded into course materials for immediate reference. Unfortunately, students often misuse on-line course materials, ignoring active links and compiling hard copies of discussions and resources, devaluing the materials' potential for active learning (Weems, 2002).

To synch or not to synch? The debate over synchronous versus asynchronous interaction continues to rage in the literature. The meta-analysis conducted by Bernard et al. (2004) revealed that supporters of asynchronous instruction cite the "anytime, anywhere" freedom provided by asynchronous education as well as the time allowed for students to carefully construct their own thoughts and reflect on the thoughts of others. In his lists of canons for distance education, Kubala (2000) actually admonishes users to avoid synchronous interaction such as chat rooms. On the other hand, those championing synchronous instruction cite benefits such as a more authentic interactive learning experience, promotion of community, and increased student accountability and retention. They argue the need for a synchronous component to enhance learner-teacher and learner-learner interactions. These two views are not necessarily at odds with each other, and their relevance in teaching mathematics will be compared and discussed later in this chapter.

Community and Discourse

Course structure. The most difficult and important aspect of teaching on-line was espoused not to be using technology but rather creating a sense of community and belonging. Sunderland (2002) espoused the use of e-mail to allow students a sense of community, immediacy of response, and anonymity. Harrison and Bergen (1999) recommended a "social thread" where students can share information that is not directly tied to the course. Geelan and Taylor (2001) also promoted multiple discussion threads to separate discussions of a social and technical nature from those regarding course content. Rovai (2003) found that "grading the on-line discussion motivated the students to increase the number of weekly messages they posted" and ultimately improved student engagement and learning. Testone (1999) promoted the formation of community by having students post personal introductions, and Sullivan (2001) further recommended collecting student pictures for on-line display. King and Puntambekar (2003) found that while progress toward the construction of community in courses was slow, the communities were eventually formed and were helpful in the context of the classes taught. For example, engaging students in on-line discussions required them to express their thoughts in writing, which supports critical thinking.

Discourse. Communication is one of the most critical aspects of computer-mediated instruction, encompassing interaction between student and teacher, between student and student, and between student and course tools. Lack of discourse due to distance, isolation, or poor course structure could be a primary cause of retention problems in on-line courses. Geelan and Taylor (2001) note the need for not only open discourse, but also discourse critical to the topic being studied. Discourse can be facilitated in a variety of ways, including face-to-face meetings, threaded discussions, chat rooms or forums, synchronous classes allowing two-way audio and/or video, e-mail, and instant messaging for private conversations. Some believe that even with two-way audio communication, discourse suffers when student-teacher interactions are conducted without the visual cues available through direct eye contact (Inman & Kerwin, 1999).

Face-to-face meetings. Some authors supported initial face-to-face meetings as a means to provide students with an orientation to technology and course expectations and to build a sense of community (Cooper, 2000; Huff, 2002). It was recommended that this orientation should be recorded for later reference. Sullivan (2000) claimed that lack of face-to-face interaction results in only a small percentage of students favoring on-line over on-site instruction.

Pedagogy

Learner-centered instruction. Bernard et al. (2004) conjectured that the power of distance education lies in its potential to implement constructivist-based approaches, provide effective interpersonal interaction, and create learner-centered environments. One result of this pedagogical approach is a shift in the instructor role from content leader to content facilitator (Smith et al., 2001). Interestingly, students appear to be more comfortable with this transformation than are teachers. Perhaps it is for the sake of the instructors that King and Puntambekar (2003) espoused a slow but steady transition from teacher-led to student-led instruction over the course of a semester.

Bernard et al. (2004) recommended that computer-mediated instruction should incorporate active collaborative learning experiences using approaches, such as problem-based learning, that induce collaboration among students. However, while recommendations to engage students in collaborative learning are common, students may resist these efforts. Hall and Keynes (1990) found that 75% of students in a distance education course preferred individual to group work, while only 25% participated in self-help groups.

Modes and levels of interaction. The one-on-one nature of interaction has emerged as a positive aspect of distance education. One-on-one contacts between the student and teacher may occur up to three times more often in on-line courses than in an on-site course (Geelan & Taylor, 2001; Harrison & Bergen, 1999; Testone, 1999). This is one of the paradoxes of distance education: students are geographically isolated from the instructor, but may well receive more one-on-one attention from the instructor then their classroom-based counterparts.

Smith et al. (2001) identified three types of interaction: learner-content, learner-learner, and learner-instructor. Schmidt, Sullivan, and Hardy (1994) found learner-learner interaction to be the most important in establishing a sense of community and suggested that threaded on-line discussions engage students in more learner-learner and learner-instructor interactions than in many on-site classrooms.

Workload. Lawless (2000) found that student workload is among the most significant factors affecting retention in distance courses. Sunderland (2002) concluded that distance education instructors fail to adequately account for the time required to complete on-line assignments. This failure, when combined with students' mistrust, misunderstanding, and misconceptions about course material, can lead to increased dropout rates. Sunderland's solution was to address affective issues, such as perceived caring based on continuous support and availability. Issues of high instructor expectations, student time on task, and student work load must be given serious consideration.

Assessment

Formative feedback. A management issue that appeared frequently in the literature was the need to provide timely and constructive feedback on student assignments and questions. Distance education precludes the regular contact found in an on-site course where students receive informal feedback on assignments on a regular schedule. Thus it is essential to set a schedule for responding to student assignments and questions.

Quality of instruments. Assessment issues, especially testing, elicited varied views. Cooper (2000) insisted that exams be given on campus to control cheating. Serwatka (2002) argued that distance courses must not be too place-based and predicted that future technological advances would solve many of the security concerns. As in face-to-face courses, it is essential to align on-line assessments with course objectives, which ideally should be performance- and competency-based. Evaluation tools must be designed and implemented with this alignment in mind (see Thompson (2004) for an example).

Overall effectiveness. Student performance may be poorer in on-line course assessments relative to students taking an on-campus course. Karr, Weck, Sunal, and Cook (2003) found that students in a traditional mode of instruction performed better on the in-class portion of examinations. They noted that this might have been due to the tendency of instructors to drop inadvertent hints about the test in the course of lecturing.

Content

Computer-mediated courses share many development and implementation issues with face-to-face courses, in addition to confronting unique challenges inherent in distance education. Likewise, in teaching mathematics and mathematics education courses on-line, we confront many of the general considerations outlined in Table 1. However, our experience tells us that the symbolic, iconic, abstract, and technology-supported nature of mathematics create special challenges for teaching the subject on-line. It is these issues, as well as the implications of other Table 1 elements for mathematics, that will be our focus for the remainder of this chapter.

Themes from the Literature: Mathematics On-line

So what if anything does the literature tell us about teaching mathematics in a distance learning format? Here we limit our discussion to a review of research on teaching mathematics via distance, specifically regarding computer-mediated courses offered via the Internet. In searching the literature from 2000 to 2006, we found only eight articles. Of these, Engelbrecht and Harding (2005a) provided the most pertinent information for this review, with an overview of technologies, attributes, and implications of teaching mathematics on-line. While espousing the flexibility and power of teaching on the Internet and the resultant paradigm shift to distributed learning, the authors warned that mathematics provides particular challenges due to the nature of the subject. First, mathematics is symbolic in nature and there are distinct problems in reproducing mathematical symbols in an on-line environment. Second, mathematics is conceptual by nature and concepts may be difficult to develop due to the isolation of on-line learning. Third, assessment of mathematics on-line is difficult due to the iconic, symbolic, and abstract nature of mathematics.

Engelbrecht and Harding (2005a) discuss a variety of technologies that are beginning to overcome these obstacles to teaching mathematics on-line. The advent of Mathematical Markup Language (MathML), the use of Java plug-ins to represent mathematical formulae, text editors such as the Math Type Equation Editor, converting LaTeX documents to HTML, and the NSF-supported TechExplorer are all helping to overcome the symbolism issue. In addition, virtual learning systems or learning management systems such as WebCT and Blackboard continue to strengthen their capacity to manage and facilitate courses that teach, explore, and assess the conceptual nature of mathematics.

Engelbrecht and Harding (2005b) also explored pedagogical aspects of teaching mathematics on-line. They acknowledge that constructivist and social constructivist cognitive theories underlie many of the current trends in teaching meaningful, conceptually-based mathematics. However, they warn that constructivism is hampered in an on-line environment due to students' perceived collaboration difficulties. Students are deprived of eye contact and body language, and simply converting a good mathematics lecture course to an on-line format or providing video lectures will not overcome this barrier to the social nature of teaching.

The authors go on to identify several benefits and problems of teaching mathematics on-line. Among the benefits is the wide range of mathematical resources that are available on the Internet, including searchable documents, interactive and illustrative applets and assessment tools, and exploration opportunities for students. Dynamic learning environments make it possible to have the exposition of a mathematical problem fully available to the student. Electronic writing tablets provide a means of sharing symbolic and visual representations more easily. Immediate feedback, sustained student interaction with meaningful mathematical problems, for the importance of in-depth discussion for mathematical concepts to mature, and the need to develop mathematical community are seen as essential to learning mathematics, but lacking in the on-line environment. On the other hand, since mathematics is less verbal and subjective than other subjects, debate and interpretation may play a lesser role, making the subject more amenable to on-line teaching.

Karr et al. (2003) studied the performance of students in a graduate engineering mathematics course. The students were divided into three groups: traditional instruction only, Web-based instruction only, and a mixture of Web-based instruction and traditional instruction. Overall, there was little difference in the performance of the three groups, although the researchers found that students who received the on-line mode of instruction performed better in the analytical portion of the course. They conjectured that students had to become more independent learners and dig deeper to understand material if they took the course on-line. Weems (2002) also conducted a study of the efficacy of learning mathematics on-line, but at the beginning algebra level. She found that students performed as well in an on-line version of the course as they did in a face-to-face version, though there was a disturbing downward trend in success for the on-line students. There was no difference in attitudes between students in the two courses.

Taylor and Mohr (2001) studied attitude and anxiety among students enrolled in developmental mathematics courses offered on-line in Australia. They found that using a range of student-centered strategies, including relevant in-context materials, informal language, and reflective practice through keeping a diary and essay writing, had a positive impact on students' mathematics anxiety and confidence in doing mathematics. However, Lawless (2000) found workload, a primary factor in retention, to be amplified in on-line mathematics courses due to the problem solving nature of mathematical tasks. Testone (2003) supported the need for student-centered strategies in teaching a developmental mathematics course on-line, identifying communication as a key issue. She recommended that communication be clear and concise, highly visible through frequent contact with students, and empathic. Cope and Suppes (2002) studied gifted high school students' performance in computer-based calculus and linear algebra courses. Despite the fact that the students formed a relatively homogeneous group, their performance varied significantly. They noted that one of the potential strengths of on-line courses is providing for a wide variation in mathematical ability by allowing students to repeatedly review archived presentations and to progress at their own pace.

Description of Programs

A comparison of our home institutions, West Virginia University (WVU) and Montana State University (MSU), reveals both similar and contrasting features. Each university serves a rural state, however, in the Appalachian region rural isolation is due largely to topography while the Rocky Mountain region imposes isolation through great distances. Both campuses have developed extensive programs to provide coursework and professional development to practicing mathematics teachers, and are experienced in delivering on-line instruction. However, MSU has taken a largely asynchronous approach to computer-mediated instruction, while WVU has adopted a synchronous approach, supported with a variety of communication tools and interactive software. We will exploit these similarities and differences in our discussion of teaching and learning mathematics and mathematics education in an on-line format.

The NSF Centers for Learning and Teaching

The National Science Foundation commissioned its Centers for Learning and Teaching (CLT) to enact a comprehensive, research-based effort addressing critical issues and national needs of the science, technology, engineering, and mathematics (STEM) instructional workforce. WVU and MSU are active partners in two different CLT programs. The Center for Learning and Teaching in the West (CLTW) joins Montana State University with Portland State University, Colorado State University, University of Northern Colorado, and The University of Montana. The Appalachian Collaborative Center for Learning, Assessment, and Instruction in Mathematics

(ACCLAIM) is a collaboration between West Virginia University, University of Tennessee, University of Kentucky, University of Louisville, Ohio University, and Marshall University. A significant component of both programs is the provision of on-line coursework for future educational leaders who are pursuing doctoral degrees in mathematics and science education.

ACCLAIM. The doctoral program sponsored by the ACCLAIM partnership consists of 20 courses in mathematics, mathematics education, rural sociology, and research methods. Eleven of these courses are offered on-line, with the remainder completed in three intensive five-week summer sessions. The on-line mathematics courses include Linear Algebra, Discrete Mathematics, History of Mathematics, and Advanced Calculus. The mathematics education courses offered on-line include Mathematics Curriculum, Mathematics Technology, and Advanced Studies in Mathematics Education. The on-line courses are shared between the six campuses and are supported by a common delivery platform. Centra software provides an integrated and comprehensive package of on-line tools including two-way audio and video, a virtual whiteboard, instant survey and feedback capabilities, and tools for sharing software and websites with participants. Both the Vista WebCT and the Blackboard course platforms provide individual e-mail, multiple discussions, assessment tools, and the ability to post course materials and assignments. Students in ACCLAIM courses meet synchronously with their instructors once each week for a two-hour session, supported by asynchronous contact through the discussion board. Some instructors also offer synchronous "virtual office hours" for one hour each week. ACCLAIM course designers follow a set of program guidelines to ensure consistency and quality in course development and delivery.

CLTW. The doctoral programs in mathematics and science education sponsored by CLTW are anchored by a unified on-line core curriculum, supplemented by regular coursework offered at the various partner institutions. The core curriculum is derived from a set of 15 three-credit course offerings, grouped thematically into five "triads": Curriculum, Assessment, and Evaluation; Diversity and Equity; Professional Development; Educational Research; and Public Policy. Each triad is sponsored by one of the five partner campuses and delivered using WebCT or Blackboard course platforms. Doctoral fellows at the partner campuses must complete two or more triads, depending on whether their programs emphasize content or curriculum and instruction.

CLTW distance courses do not employ the software, audio, and video tools used by ACCLAIM; instead, they rely heavily on asynchronous activities and discussion. Through carefully structuring and facilitating text-based discourse, instructors seek to create an environment where "members question one another, demand reasons for beliefs, and point out consequences of each other's ideas—thus creating a self-judging community" (Garrison et al., 2001, p. 12). A typical course may include both scaffolded and open-ended discussions, completion of readings and activities, and presentation of research in on-line forums. CLTW does not mandate a particular course design framework; instructors are free to sequence and facilitate learning activities in ways that suit their own preferences and experience. However, each triad was collaboratively designed, and successful practices and "lessons learned" are shared regularly among CLTW faculty.

Coursework for Practicing Teachers

Our institutions also provide a number of on-line mathematics and mathematics education courses outside of the Centers for Learning and Teaching. MSU offers a longstanding and very successful Masters of Science degree in Mathematics Education that includes a substantial on-line component and enrolls students from across the United States. WVU, in partnership with the South Regional Education Board, has created a series of on-line courses for middle school teachers in sixteen states. The audience for both of these programs is primarily made up of practicing teachers of mathematics, grades 6-12, who are pursuing graduate coursework.

SREB/MERIT. This project, originally supported by an NSF-funded Mathematics Education Reform Initiative for Teachers (MERIT), targeted the implementation of model curricula in middle school mathematics across the state of West Virginia. MERIT was a substantial professional development effort supporting lesson study and the development of learning communities to address change in teaching mathematics in middle school. Early in the program it became evident that some teachers needed a stronger content background in mathematics to effectively implement NSF-funded standards-based curricula such as Math Scape or Connected Mathematics. MERIT formed a partnership with West Virginia University and Marshall University to develop 24 hours of on-line coursework for middle school mathematics teachers. The courses were organized into four six-hour course cadres, integrating four credit hours of mathematics content with two hours of mathematics education. Two of the cadres, Number & Algebra and Functions & Calculus, were offered through West Virginia University, while the remaining two cadres, Geometry and Data Analysis & Probability, were offered through Marshall University. More recently, West Virginia University has partnered with the Southern Regional Education Board (SREB) to offer the Number & Algebra course and the Functions & Calculus course to a sixteen-state region in the southeastern United States.

The Number and Algebra cadre was originally offered in an audiovisual format using lectures recorded using Tegrity software. The Tegrity sessions could be viewed live or taped but lacked a good mechanism for two-way communication, so the only truly synchronous component was provided by a WebCT-based chat room. For the past two years Centra has served as the software platform for both the lessons and virtual office hours. Centra has enabled much more interactive synchronous communication, including two-way audio transmission during the live sessions.

MSMME. The Mathematics Option of MSU's Master of Science degree in Mathematics (MSMME) is designed for secondary or junior college teachers of mathematics. The program attracts teachers of grades 9-14 interested in a content-rich masters degree focusing on the effective teaching and learning of mathematics. MSMME is specifically designed to reach teachers-in-service where they live and work. Mathematics courses, designed to challenge teachers' content knowledge in areas that are relevant to school mathematics, include Analysis, Linear Algebra, Geometry, Statistics, Number Structures, Technology, and Mathematical Modeling. The mathematics education courses address learning theory, standards-based instruction, assessment, and the language and history of mathematics. With well over one hundred successful graduates since 1991, the program has influenced mathematics instruction throughout Montana, across the United States, and as far away as Alaska and Hong Kong.

Currently, two on-line courses are offered each semester (both academic year and summer). All on-line courses are offered via WebCT courseware in an entirely asynchronous format. The courses are text-based, without support from presentation tools or communication software. However, several courses such as Geometry, Technology, and Mathematical Modeling routinely require participants to perform complex operations using calculators and other technology, use external software such as Geometer's Sketchpad, and post mathematical assignments for others to view.

Face-to-face summer courses are offered in a compressed three-week institute format. Most of the program can be completed at a distance; however, students are required to be on campus at least one summer to complete a research course in preparation for their capstone project. The capstone, an action research project addressing a problem or topic of interest identified in the teacher's classroom, school, or district, is intended to give participants the opportunity to synthesize a significant body of knowledge based on their MSMME coursework. The results of the project are presented in a face-to-face summer symposium, followed by a comprehensive oral examination.

Mathematics On-line: User Perspectives

We now turn to an examination of issues and perspectives shared by students and instructors in the four on-line programs sponsored by West Virginia University and Montana State University. The concerns and affirmations expressed by our course designers, instructors, and consumers mirror many of the issues expressed in the general literature regarding on-line learning. We will present these perspectives in alignment with the themes identified in Table 1.

The student responses summarized and reported here were collected through qualitative interviews and surveys. Different interview and/or survey protocols were used for the ACCLAIM, CLTW, MSMME, and SREB groups. However, the goals of the instruments were similar—primarily to investigate positive and negative aspects of participation in on-line coursework and distance-based programs. The findings reported here represent a compilation of input from more than 50 students engaged in graduate-level coursework in mathematics and mathematics education.

Besides mining the existing data from students, we conducted formal interviews with eight university faculty members who are primary instructors in the four distance-based programs. We used a semi-structured interview protocol with questions and prompts based on the themes and sub-themes portrayed in Table 1. The faculty members, all from mathematics departments, were asked to comment on how these themes play out in their on-line courses. They also shared what they perceive to be obstacles and benefits to teaching and learning mathematics in an on-line environment.

It is possible to analyze this rich body of data from several perspectives. We might compare those who see a significant difference between on-line courses and classroom-based work with those who find the difference to be minimal or non-existent. Alternatively, we can compare the views of those in high-technology courses with those in text-based courses, or simply compare student and instructor perspectives. To avoid creating a false sense of "right vs. wrong" in any of these dichotomies, our discussion will be structured to align with Table 1, and we will present all views under each theme collectively.

It is important to remember that these insights are limited to participants in on-line graduate courses for practicing teachers and motivated educators. For this audience, there is no realistic option for a residential program. Furthermore, students in these programs generally demonstrate a high degree of maturity, organization, and focus. In the words of one faculty member:

> I think the structure of our courses is really a function of our audience. We have a responsible audience…they enter the courses with a collaborative mindset…they want to

> share and learn from one another….I would do it much differently in an undergraduate math course.

Audience

As in many distance learning scenarios, students in our programs consistently cite opportunity and convenience as strengths of on-line learning. Montana's masters program enrolls teachers from across the nation and as far away as Alaska and Hong Kong. Teachers such as these are looking for high-quality programs that are aimed at improving their own teaching, and are willing to select a distance program that serves their needs over a local generalist program. In both Montana and West Virginia, rural isolation precludes access to campus-based programs during the academic year, and on-line coursework offers an attractive option for pursuing an advanced degree. Even for those within reach of university campuses, the ability to pursue an advanced degree while maintaining a full-time teaching career and remaining with family is an appealing choice.

Some students claim that the on-line learning environment is less stressful than a classroom environment, enabling them to learn and digest material more comfortably. Others are challenged by the limits on communication and wish for more visual contact. Instructors are concerned about meeting their students' needs: "In some sense the toughest part of the course wasn't so much the on-line component but the range of skills of students in the cohort. Some had really weak backgrounds. There were a variety of strengths."

Instructors agree that learning styles and motivation levels vary widely among their on-line students and need to be addressed, whether this means encouraging a quiet student or "shutting down" an overly assertive class member. They view self-motivation and the ability to work independently as a characteristic of success, noting that the most successful students are "proactive learners" who make thoughtful and extensive contributions to course discussion. In a classroom situation, "You can get by with being silent in the class. In an on-line course, you just have to take a chance…[you're] being graded on talking." One instructor posits that the difference between on-line and classroom-based courses is less significant than the differences among students and their motivation levels. Another observes:

> Maybe the audience issue has more to do with the audience for learning mathematics rather than the audience for learning on-line….The same difficulties they have learning mathematics in the classroom become on-line issues. Whether they are quantitative thinkers, viewing the world quantitatively…how good they are at abstract reasoning.

Instructors of discussion-rich courses feel that natural leaders often emerge within a group, modeling good interaction skills and eventually drawing others into the conversation. For instance, in a problem-based mathematics modeling course, "You need a critical mass of students…first that are capable, and [then] willing to share their ideas….Verbal leaders…who are going to get the ball rolling, comment on other peoples' work."

Medium

The on-line courses offered by Montana State University use a WebCT platform and rely heavily on text-based discussion, sharing of attachments and posted documents, access to Web resources, and technological tools such as uploaded calculator data, applets, and software. By contrast, West Virginia University minimizes text-based presentations and discussion in favor of enhanced audiovisual approaches made possible by Centra and Vista WebCT technology. These differences in delivery are reflective of another key difference. Montana courses are almost entirely asynchronous, with an occasional real-time component of "live office hours" based in a chat room. By contrast, West Virginia courses meet regularly once or twice a week for live interaction, and asynchronous discussion is limited to a support role or is nonexistent. These contrasting approaches to teaching and learning naturally result in contrasting views of technology and synchronous vs. asynchronous learning, which we address at the end of this section.

All four programs build extensive use of Web-based resources into their course designs. Instructors provide URLs for historical information, definitions and examples, and mathematical proofs. Students conduct Web research for presentations on assessment and standards. They explore interactive applets to experiment with projectile motion. Statistics students collect live data on eruption cycles in Yellowstone National Park, then use an on-line statistical software package to analyze and interpret the data. Many of these activities would be difficult and inefficient to enact in a classroom setting.

In many courses, textbooks have been replaced by materials posted by the instructor, located on the Internet, and available through electronic reserve at university libraries. In place of textbooks, students may be asked to purchase a software package or calculator that supports exploration and sharing of results. One instructor requires students to purchase a specific calculator and graph link system: "That allows them to easily incorporate graphs, tables, data, equations—it allows them to show me their work. If they write programs, they can send me the program." Students in a geometry course use Geometers Sketchpad software to explore concepts and demonstrate proofs, and as a tool for sharing sketches and mathematical arguments. These applications are a powerful support in both synchronous and asynchronous courses. "We had the ability to deliver multimedia…audio/video, anything that could be on-line….We could even vote on things [and] display a bar graph of results."

The WVU programs have outdistanced MSU in experimenting with audio and visual technology for on-line teaching as well as the use of electronic tools. Live sessions supported by two-way audio and video communication are scheduled on a weekly basis. In some courses the live sessions, both instructional and tutorial, comprise the majority of a course, with asynchronous discussion and sharing of material playing a secondary role. Several instructors in the WVU programs express a desire for even greater audiovisual interactivity. "I could see everybody's faces if they all had a camera—so far they don't…. I can't see puzzled looks or "aha" looks….Being able to see each other really does help a lot."

Even those with fully functional two-way contact are not completely satisfied. Instructors still see limitations in the video technology. "I couldn't see the students' faces as I was talking so it was hard to sense whether or not they were understanding what I was saying as I was saying it." Students shared the same feeling: "I just think there's something lacking when you…don't see people face to face. I don't think the instructor…grasps…to the same extent whether or not everyone's on the same page."

Some instructors create PowerPoint notes to supplement lessons, and many record and archive their video lectures. Such practices allow students to prepare for the lesson: "His notes that he sent us ahead of time helped me when I listened to the lecture…to be able to take notes." Students also know they can access and review lecture material later, making the actual instructional session more comfortable. "I just feel more relaxed, I don't feel like I am going to miss as much…you could listen to it at your own pace and you could go back and review it."

Instructors generally find that these tools enhance their ability to display mathematical representations, to demonstrate examples, and to help students visualize mathematical concepts. In this highly interactive environment, they can create a page of work on a whiteboard while conducting a question and answer session, and then save and send the notes to their students. Some instructors who have taught asynchronous courses in the past find the added technology makes a significant improvement; one noted that it allows him to "do things in front of them like in a regular classroom."

Students in WVU courses report that the whiteboard and live sessions make symbolic manipulation and mathematical discourse less burdensome. They feel that the combination of media gives them a greater sense of being in a traditional classroom. Initial awkwardness with the tools is mediated by pre-course training sessions and help from classmates. However, use of these tools gives rise to a different sort of burden based on hardware limitations. For example, the whiteboard application is capable of displaying student as well as instructor input, but not all participants possess the necessary technology. The whiteboard's limited writing area also makes it difficult to process a lengthy explanation or proof that can only be displayed one portion at a time.

The fundamental challenge to high-technology course tools remains the disparities in hardware and Internet connections used by the student audience. Features intended to simulate real-time interaction are sometimes confounded by connectivity delays. Rural teachers in particular are limited to dial-up connections and are frequently frustrated by connectivity problems. "Our rural connections are not as fast as the DSL… it can be frustrating if you get cut off….This got in the way of learning the material…you can get kicked off like nothing." In one student's view, "It takes too long to ask questions…and part of the time you can't hear them, so they have to repeat the question. I don't think you're getting as much accomplished in the class time because the technology is in the way."

Instructors feel the same frustrations as "students would get thrown offline by their servers—people would kind of fade in and out of the class." They suggest that future courses include minimum technology requirements so that all students will have the capability to fully participate. At the same time, they believe that "as technology becomes more available those limitations will be overcome." Instructors at Montana State University wholeheartedly agree that technology will soon make it easy to communicate through audio, visual, and symbolic media. However, the limits created by widely varying levels of access have caused them to avoid using some interactive media tools. The perspective at MSU is well represented by one instructor's words:

> I think there's going to be a time for change….Until there's equity and everybody in the class has the capabilities of everybody else, it limits the desire to make too much of a move. Things like the whiteboard, those can be nice—but if I'm able to write on the whiteboard, now I have provided a disadvantage to students who can't write me back that way….Until everyone's on the same page, I don't think we're limiting ourselves.

Views about synchronous vs. asynchronous learning also differ between the two campuses. WVU instructors embrace technology-supported synchronous sessions, considering this model "like night and day" compared to the model still used in Montana. However, MSU instructors are not yet prepared to move to a synchronous model. This is in part due to the nature of their audience, which includes teachers from Alaska to Vermont and even overseas. "For one person it's midnight…for me it's 4-5 a.m.…Until I could see that a synchronous meeting could actually be more efficient than asynchronous, I'm not motivated." A second view recognizes the hectic schedules held by practicing teachers. "Lots of times they teach all day, they try to be with their kids all evening, and they're on-line [at night]….You can't pick those hours."

A third argument is that "the process of learning mathematics doesn't demand synchronous interaction." Instructors argue that allowing time to reflect on the mathematics, to explore problems as individuals, and to digest the ideas of others enables rich cognitive outcomes. Students need to come "face to face with the phenomenon of mathematics," with opportunities to follow up hypotheses, question ideas, and draw and defend conclusions. "Doing mathematics out of a book is not doing mathematics….By the end of the term, they are all building confidence in themselves as mathematicians."

Community and Discourse

Regardless of the level of technology in an on-line course, a primary objective is to establish a sense of community that will engage students and encourage discourse about the subject matter. This is not an easy task. While students may feel more confident about speaking up from the comfort and anonymity of their computer space, they may not know how to go about it. "How do you get the conversation going?" asked one instructor. "The abstract nature [of mathematics] is difficult to discuss with typing."

As reflected in the general literature, the power of discussion is a key feature of learning mathematics on-line. "You have to sort out your thoughts before you write them down. You get more one-on-one attention in an on-line course." MSU's on-line courses, often with an enrollment of twenty or more, rely heavily on text-based interaction and implement a variety of strategies for managing discussions. In a modeling course, students tackle problems in groups of four: "Helping each other is not an option, it's a requirement…these are hard problems, they need each others' help. I urge them to eavesdrop." Students in a statistics course work in larger groups, but police themselves by keeping track of topics and threads. Assessment students form groups of six with an assigned facilitator who provides prompts, raises questions and directs the flow of discussion.

Well-orchestrated discourse promotes collaboration among students as well as a sense of community. Working together, learners begin to break down the barrier "that mathematics is hard, that mathematics can't be understood, that you have to have certain abilities to do mathematics….When you are working as a group sometimes students help other students. Sometimes students understand better from other students." Students concur, noting that "it was nice to read what other people had done…it gave you somebody else's perspective, and sometimes people came from completely different directions."

K-12 teachers are natural collaborators, and part of their culture is collecting ideas from colleagues. That sense is evident in their comments. They want to share assignments, modules, and projects, noting that "We could have seen what others were doing and perhaps gotten feedback on what might help us when we got to the later stages—shared each others' successes and frustrations." Despite their positive view of collaboration, students in both states are not in favor of requiring group work in an on-line environment. They become frustrated by the level of coordination needed to work with classmates to complete a project outside the structure of the course, as well as by unequal levels of effort by group members.

Involvement in grants and graduate programs has provided opportunities for some students to meet face-to-face prior to taking a course or at least during the process of completing their graduate program. For many, the initial face-to-face contact smooths the way for on-line interaction: "We were together all last summer…so when they were on-line, they were not strangers to me." Occasionally instructors go to great lengths, even traveling to meetings in other states, to meet at least some of their students firsthand. Those who must meet their students for the first time on-line have mixed feelings about the social implications:

> Having a cohort of students had some advantages in that they all know each other, and some disadvantages in that there may be more collaboration on homework assignments….They spent enough time together that they were a well knit group. I appeared distant, and this was exacerbated by them being so close. That exaggerated all the effects of teaching at a distance.

Pedagogy

The experience of the two universities demonstrates that mode of delivery greatly influences mode of instruction. An instructor notes: "[My approach is] very interactive, more of a constructivist view. Get students to come up with ideas on their own before presenting them to the group. Lots of give and take, but I do provide information." A learner-centered model is not only effective, but efficient in the on-line instruction model.

> What I do is I pose problems…they basically solve the problems either individually or in groups—discuss them…and then they submit a solution. I don't do a lot of lecture.
>
> I have a series of problems that are expanded into activities (one per week)… followed with questions that evoke difficult problems and demand a rethinking of basic mathematics…sometimes I ask more specifically that they prove something…I like them to feel that they can give a compelling demonstration.

Asking students to solve a problem is easy; getting them to clearly communicate and compare their approaches and solution strategies may require prompting. One instructor's solution is to have everyone submit an initial response to the problem as an official record of their thinking before they read other solutions. "Then they read the others' comments or work [and] have to respond to at least three or four." Another notes that, "If you pose a good challenging problem, they automatically get in groups and discuss it. You can also have them review work more easily—it's almost like a public forum….It's very easy to get them working collaboratively." Research indicates that communicating about mathematics and working with others on mathematics problems are strategies that enhance understanding and retention of concepts.

As noted earlier, an interactive and problem-based approach to on-line mathematics learning minimizes the need for a formal textbook. "In the history of mathematics course, [we use] a history topics book…in modeling, we just have readings—you can basically create a text via the e-reserves." Depending on the content of the course, better results may be produced by tailoring course materials to individual needs. "They each have the [statistics] book…that's kind of like their handbook, and the place where most of their individual problems come from. What I have on-line is my 'lectures'…topics, projects, problems…I do projects that pull the textbook stuff together."

All of the on-line courses referenced here rely to some degree on discourse as a vehicle for teaching and learning. Just as instructors are aware that they must carefully balance an emphasis on technological tools with the ability of the students to use them, they must also take care that discussion threads and postings are meaningful and of high quality. Students note that "one of the best attributes of a good on-line course has been really good discourse." They liken discussions to "guided inquiry" and observe that "discussion is one of the most valuable things, but if that instructor can facilitate that discussion to allow, or cause, everyone to be [involved] and *learn* as much as they can…I think it's critical."

Students are aware and appreciative when instructors skillfully facilitate course dialogue. They single out those who provided "well-chosen questions" to encourage reflection about readings and are "responsive and thoughtful as he pushed and supported students with great differences in their background knowledge and experience." One student said: "The instructor didn't necessarily post a message every day, but I was certain he was following everything and offering guidance as necessary….He kept a number of different threads going at the same time without overburdening his students. I always felt valued." They also recognize their own responsibility in creating quality discussions: "[Good] discourse has come from each participant's own center of wanting to share and gain from others, and ask questions and offer questions." They admit their own tendencies to digress from the topic at hand. "People are putting up an opinion, and the next opinion might have nothing to do with the previous one. So it's not a discussion, but a series of monologues." In other situations, "Somebody focuses on a really insignificant point…the instructor needs to know how to step in, redirect the conversation."

Little was said about workload and length of assignments, but students are adamant about the importance of prompt and useful feedback. This includes regular participation and feedback in on-line discussions, as well as prompt feedback on student assignments. Students want formative comments to help them improve their understanding; at times, they simply want evidence that their work has been received and assessed. One student "was panicking on one test I took because…I didn't get any feedback telling me he had actually gotten it." Instructors agree: "They like getting individual responses, they like to know their work is being reviewed and valued…The last thing [they] want is to send something into this distance black hole."

Students and instructors alike also indicate that organization is particularly imperative in the on-line setting. This includes providing a course syllabus and explicit instructions, setting clear expectations for participation in on-line discussions and assignments, and setting deadlines for completing problems and responding to prompts. Some instructors post all course materials—topics, readings, and assignments—up front. However, students tend to take posted material to heart, limiting future flexibility. "You really can't effectively change something once it's up and running….add a new homework problem…switch what's required and what's due." With that view in mind, some instructors choose to reveal only one lesson or unit at a time

In keeping with the current literature, instructors at WVU are concerned that the advanced technology supporting their courses may also diminish their effectiveness. Notes one instructor: "The more powerful a tool becomes, the easier to teach the same old way. There is strength in these powerful packages, but also the danger that [we] will fall back on writing on a whiteboard all the time." Another observes, "It is very tempting to say, Centra is great because you can do the same thing in Centra as in the classroom, but this might not be the right thing to be doing in the classroom." Without visual "proof" of student engagement, instructors also worry about having their students' full attention. "I don't know how much electronic note passing goes on behind the scenes….[There is] a temptation to multitask." Students agree: "I felt that it was very easy for students not to participate in class. I thought that people could log on, and then, whatever, watch television for an hour."

Assessment

Many assessment issues that arise in mathematics courses are similar to those in other disciplines. Academic honesty is a concern for some instructors: "I guess I'm never 100% sure that the person who's on the other end of the computer is actually the person they say they are…Quizzes and tests are more challenging. You don't know if the students have their books open…you can't proctor….So you have to be a little creative." One instructor wasn't sure how to adapt his traditional take-home finals to account for potential collaboration. Another resolves that dilemma by mailing the exam to students with instructions to open the envelope, spend no more than one hour on the exam, and send it back. Overall, instructors feel confident that student work on exams is consistent with their coursework, and comparable to progress demonstrated in classroom-based courses.

There is consensus that assessment of mathematical thinking takes on new characteristics in an on-line environment. Necessity has given rise to a surprising variety of on-line assessment models. For example, MSU instructors often grade discussion contributions as a means of day-to-day formative assessment. "You see their entire dialogue… I can get an idea of where Susie's at. She has to speak because she's graded on speaking." Students take traditional quizzes and exams, whether they are administered electronically or in hard-copy form. Solved problems, homework, and written summaries are often graded using a rubric, and corrected work is sometimes accepted for resubmission. In addition to final exams, projects and presentations may be assigned as summative assessments. One instructor requires students to create a project where they explore a new use of modeling in mathematics, explaining the context, the content, and connections to the classroom. Another collects a series of three problem portfolios throughout the semester, including explanations, demonstrations, and proof. Such extensive bodies of work need not be electronic; students sometimes fax or mail handwritten materials.

Instructors from all programs agree that timely and individualized feedback is an essential component of assessment, whether it takes the form of comments on an assignment or task, responses posted to a discussion thread, or personal e-mails. Several are still seeking reasonable methods of giving feedback on students' mathematical work, or for helping them work collaboratively. "Getting students to write proofs…and getting them to improve their proof writing is a challenge….I don't think we have a good solution. Not until we can come up with a way for people to write mathematics on a common virtual sheet of paper."

Issues Specific to Mathematics

Mathematical symbolism

> The difficulties in assessing mathematics on-line are the same ones that make it difficult to teach, the abstract nature of the topic, also the symbolism and the visual nature. How should they submit these things on-line?

These words express the challenges that confront the designers and facilitators of any on-line mathematics course. Electronic whiteboards and video cameras that allow demonstrations are of great benefit to instructors, but students still struggle to represent symbols in their typed homework. Even with the availability of equation editors and other similar tools, students are unhappy with the time required to learn the software and create symbolic expressions.

Instructors and students have been resourceful in finding ways to circumvent this barrier. One instructor insists that "symbolism just hasn't been an issue" and explains how some of her students have developed a "class lingo" rather than taking the time to open and use a symbolic editor: "They will just write the word 'x bar'…for the sigma sign, they just write 'sigma.'"

Other forms of technology can be "borrowed" to reduce the burden of representing mathematics in symbolic form. In some courses, students write proofs and create demonstrations using Geometer's Sketchpad, a software package familiar to many teachers that allows them to easily compile sketches, graphs, mathematical expressions, and text, then share their end product as an attachment or embed it in a Microsoft Word document. Some assignments send students on a Web search to find solutions to classic problems in mathematics. "Believe it or not…the proof is on-line somewhere….the Web is a big resource."

Many graphing calculators have the capability to allow data and images to be uploaded to a computer and inserted into other documents. However, unless all students are required to purchase the same calculators and accessories, this option is not accessible to everyone. As an alternative, students can create and capture a variety of statistical and algebraic graphs using applets and programs that reside on the Web. Instructors may also pose problems and assign activities that require students to create and analyze graphs without having to transmit them.

One instructor from MSU, where most work is still text-based, notes that "If I was to teach [discrete mathematics] where a lot of the ideas are unfamiliar to them…I want to get a whiteboard. I think I would probably do more and be able to interact better with the students." But he also points out that the teacher audience in graduate mathematics coursework is probably "at least vaguely familiar with the mathematics" and better able to accommodate for the limitations inherent in on-line communication.

Mathematical technology

> I try to do everything possible to minimize the negative effect of technology and sort of provide equity to the students….I think they're most happy that the technology isn't limiting their learning….It shouldn't be an issue. Technology is a tool, it's not the objective.

Instructors are well aware that the best-laid plans can go awry when students are learning mathematics at a distance. Web sites go down; data sets get corrupted; calculators and software refuse to cooperate. Instructors offer Web-based help sites, tutorials, programming scripts, and pre-designed examples to help students become comfortable with the technology required in their courses. Assignments—and expectations—are slowly scaled up as students become more familiar with tools and software. One instructor in a calculator-heavy course asks his students to purchase a specific calculator in place of a textbook. He can then better focus his efforts on providing support. "I create a lot of "how-to" Web pages on using the technology…keystroke specific. I have a technology "help center" [discussion]….I tell everyone…if you can help the student, help them….It builds more trust, and I'm not having to assume all the responsibility."

Overall, instructors in the WVU and MSU programs have a healthy attitude about working with mathematics students on-line. "I'm more worried about if they're correct, and less worried about whether everything is lining up neatly….I'm not trying to teach them how to… write symbols in a Web-friendly environment…. The purpose is interacting with the mathematics, not learning how to be more Web proficient." Course designers are aware that not all students can invest in the newest technology. Using text-based "lingo," scanning documents, and even mailing materials are acceptable alternatives to using advanced technology. An instructor notes, "Technology shouldn't be a hindrance…if worst comes to worst, I say write out your solution and fax it to me. If we spent all our time learning to use equation editors, that would be the tail wagging the dog."

Conclusion

So what do we conclude about teaching mathematics or mathematics education on-line from the literature, research, and empirical experiences at MSU and WVU? First, teaching mathematics or mathematics education involves difficulties that are inherent in the nature of the subject; in particular the symbolic, abstract, and visual nature of mathematics. Second, there are varied approaches to addressing these problems which are based more on practical experience then research based practice. Thus there is a need for extensive research on the learning and teaching of mathematics and mathematics education on-line. Third, ever improving technology provides solutions to some of the inherent problems in teaching mathematics on-line; however, issues of equitable access and ability to implement the technology can be counterproductive. Finally, there is a great deal of potential in teaching and learning mathematics and mathematics education on-line for the willing student and savvy instructor.

References

Allen, Mike, Bourhis, John, & Burrell, Nancy (2002). Comparing student satisfaction with distance education to traditional classrooms in higher education: a meta-analysis. *American Journal of Distance Education, 16(2),* 83-97.

Abrams, Gene & Haefner, Jeremy (1998). S.H.O.W.M.E.: Spearheading online work in mathematics education. *T H E Journal, 25*(10), 53-55

Bennett and Bennett (2002). Assessing the quality of distance education programs: the faculty's perspective. *Journal of Computing in Higher Education. 13*(2), 71-86.

Berge, Zane L. (2001). Review of research in distance education. *American Journal of Distance Education, 15(3),* 5-19.

Bernard, Robert M. & Abrami, Philip C. (2004). How does distance education compare with classroom instruction? A meta-analysis of the empirical literature. *Review of Educational Research, 74(3),* 379-439.

Burge (1994) Learning in computer conferenced contests: The learners' perspective. *Journal of Distance Education, 9*(1), 19-43.

Carnevale, Dan & Olsen, Florence (2003). How to succeed in distance education. *Chronicle of* Higher *Education, 49*(40), A31-A33.

Charp, Sylvia (2000). Distance education. *T H E Journal, 27*(9), 10-12.

Cavanuagh, Catherine S. (2001). The effectiveness of interactive distance education technologies in K-12 learning: a meta-analysis. *International Journal of Educational Telecommunications, 7(1),* 73-88.

Chickering, Arthur & Gamson, Zelda. (1987, March). Seven principles for good practice in undergraduate education. *AAHE Bulletin*, 3-7.

Cooper, Linda (2000). Online courses. *T H E Journal, 27*(8), 86-92.

Cope, Eric W. & Suppes, Patrick (2002). Gifted students' individual differences in distance-learning computer-based calculus and linear algebra. *Instructional Science, 30*(2), 79-110.

Davies, Timothy Gray & Quick, Don (2001). Reducing distance through distance learning: the community college leadership doctoral program at Colorado State University. *Community College Journal of Research and Practice, 25*(8), 607-620.

Englebrecht, Johann & Harding, Anise (2005). Teaching undergraduate mathematics on the internet. *Educational Studies in Mathematics, 58(2),* 253-276.

Englebrecht, Johann & Harding, Anise (2005). Teaching undergraduate mathematics on the internet. *Educational Studies in Mathematics, 58(2),* 235-252.

Garrison, D. Randy, Anderson, Terry, & Archer, Walter (2001). Critical thinking, cognitive presence, and computer conferencing in distance education. *American Journal of Distance Education, 15(1),* 7-23.

Geelan & Taylor (2001). Embodying our values in our teaching practices: Building open and critical discourse through computer mediated communication. *Journal of Interactive Research. 12*(4).

Gilbert, Janna (1999). But where is the teacher? *Learning and Leading with Technology, 27*(2), 42-44.

Hall, James (1990). Distance education: Reaching Out to Millions. *Change*, *22*(4), 48.

Harmon and Dorman (1998). Enriching distance teaching and learning of undergraduate mathematics using videoconferencing and audiographics. *Distance Education 19*(2). 299-318

Harrison & Bergen (1999) Mathematics for the liberal arts student: Pedagogical issues and strategies for a successful distance learning course. *Mathematics and Computer Education, 33*(1), 52-61.

Hawkes, M. (2001). Variables of interest in exploring the reflective outcomes of network-based communication. *Journal of Research on Computing in Education, 33,* 299-315.

Heerema, Douglas L. & Rogers, Richard L (2001). Avoiding the quality/quantity trade-off in distance education. *T H E Journal, 29*(5), 14-19.

Huff (2002). Technical skills required in distance education graduate courses. [Online]. Available: http://www.usdla.org/html/journal/SEP02_Issue/article03.html *U S D L A Journal. 16*(9),

Hummel, Hans G.K. & Smit, Herjan (1996). Higher mathematics education at a distance: the use of computers at the Open University of the Netherlands. *Journal of Mathematics Science Teaching, 15*(3), 249-265.

Inman, Elliot & Kerwin, Michael (1999). Instructor and student attitudes toward distance learning. *Community College Journal of Research and Practice, 23*(6), 581-591.

Jung, Insung & Rha, Ilju (2000). Effectiveness and cost-effectiveness of online education: a review of the literature. *Educational Technology, 40(4),* 57-60.

Karr, Weck, Sunal, & Cook (2003) Analysis of the Effectiveness of Online Learning in a Graduate Engineering Math Course. *The Journal of Interactive Online Learning, 1*(3), 1-8.

King & Puntambekar (2003) Asynchronously conducted project-based learning: Partners with technology. *International Journal on E-learning, 2*(2), 46-54.

Kubala, Tom. (1998). Addressing student needs: Teaching on the internet. *T.H.E. Journal, 25*(8), 71-74.

Kubala, Tom (2000). Teaching community college faculty members on the internet. *Community College Journal of Research and Practice, 24*(5), 331-339.

Larson, Matthew R. & Bruning, Roger (1996). Participant perceptions of a collaborative satellite-based mathematics course. *The American Journal of Distance Education, 10*(1), 6-22.

Lawless, Clive (2000). Using learning activities in mathematics: workload and study time. *Studies in Higher Education, 25*(1), 97-112.

Lewis, Laurie: Snow, Kyle; Farris, Elizabeth; & Levin, Douglas. (1999). *Distance education at postsecondary education institutions, 1997-98* (NCES-2000-013). Washington, DC: National Center for Education Statistics. (ERIC Document Reproduction Service No. ED437879)

LePage, Denise. (1996). Distance learning complements a pre-service mathematics education model. *T H E Journal, 24*(1), 65-68

Lou, Yiping (2004). Understanding process and affective factors in small group versus individual learning with technology. *Journal of Computing Research, 31(4),* 337-369.

Lou, Y., Dedic, H., & Rosenfeld, S. (2003). Feedback model and successful e-learning. In S. Naidu (ed.), *Learning and Teaching with Technology: Principles and Practice* (pp.249-260). Sterling, VA: Kogan Page.

Lou, Y., & MacGregor, S.K. (2002). *Enhancing online learning with between group collaboration.* Paper presented at the Teaching Online in Higher Education Online Conference.

Luebeck, Jennifer & Bice, L.R. (2005). Online discussion as a mechanism of conceptual change among mathematics and science teachers. *Journal of Distance Education, 20(2).*

Machtmes, Krisanna & Asher, J.William (2000). A meta-analysis of the effectiveness of telecourses in distance education. *American Journal of Distance Education, 14(1),* 27-46.

Merisotis, Jaime & Phipps, Ronald. (1999). What's the difference? Outcomes of distance vs. traditional classroom-based learning. *Change, 31*(2), 12-17.

McCollum, Kelly (1997). A professor divides his class in two to test value of on-line instruction. *The Chronicle of Higher Education, 43*(24), A23.

McKnight (2004) Virtual necessities: Assessing online course design. *International Journal on E-Learning*. *3*(1), 5-10

McKnight, C.B. (2001). Supporting critical thinking in interactive learning environments. *Computers in Schools, 17(3-4),* 17-32.

Moore, Michael G. (2001). Review of research in distance education. *The American Journal of Distance Education, 15(3),* 5-19.

Moore, Michael G. & Thompson, Melody M. (1990). The effects of distance learning: a summary of literature. Research monograph number 2. *Southeastern Ohio Telecommunications Consortium, Information & Technology.*

O'Malley, Claire & Scanlon, Eileen (1990). Computer-supported collaborative learning: problem solving and distance education. *Computers and Education, 15*(1-3), 127-136.

Parker, Angie. (2003). Identifying predictors of academic persistence in distance education. [Online]. Available: http://www.usdla.org/html/journal/JAN03_Issue/article06.html *USDLA Journal*. *17*(1),

Perez, Stella & Foshay, Rob. (2002). Adding up the distance: can developmental studies work in a distance learning environment? *T H E Journal, 29*(8), 16, 20-22, 24.

Rovai (2003). Strategies for grading online discussions: effects on discussions and classroom community in internet-based university courses. *Journal of computing in higher education,* 15(1), 89-107.

Russell, Thomas. (1999). *The no significant difference phenomenon*. Raleigh, NC: University of North Carolina Press.

Ryan, Walter F (1996). The effectiveness of traditional vs. audiographics delivery in senior high advanced mathematics courses. *Journal of Distance Education, 11(2),* 51-54,.

Saba, Farhad (2000). Research in distance education: a status report. *International Review of Research in Open and Distance Learning, 1(1).*

Sbiek, Rose Mary & Foletta, Gina M (1995). Achieving standards in a fiber optic mathematics classroom. *Learning and Leading with Technology, 22*(8), 25-26, 28-29.

Schlosser, Charles A. & Anderson, Mary (1994). Distance education: review of the literature. *Information & Technology.*

Schmidt, Kathy J, Sullivan, Michael J. & Hardy Darcy Walsh (1994). Teaching migrant students algebra by audioconference. *The American Journal of Distance Education, 8*(3), 51-63.

Serwatka, Judy A (2002). Improving student performance in distance learning courses. *T H E Journal, 29*(9), 46-51.

Shachar, M. & Neumann, Y, (2003), Differences between traditional and distance education academic performances: a meta-analytic approach. *International Review in Open and Distance Education, retrieved October 30, 2003,* from http://www.irrodl.org/v4.2/shachar-neumann.html

Short, Nancy M (2000). Asynchronous distance education. *T H E Journal, 28*(2), 56-64.

Smith, Glenn Gordon, Ferguson, David & Caris, Meike (2001*). T H E Journal, 28*(9), 18-25.

Sullivan, Patrick (2001). Gender differences and the online classroom: male and female college students evaluate their experiences*. Community College Journal of Research & Practice, 25*(10), 805–819.

Sunderland, Jane (2002). New communication practices, identity and the psychological gap: the affective function of e-mail on a distance doctoral programme. *Studies in Higher Education, 27*(2), 233-246.

Szule, Paul (1999). Reassessing the assessment of distance education courses. *T H E Journal, 27*(2), 70-74.

Tabs, E.D (2003). Distance education at degree-granting post-secondary institutions: 2000-2001. (NCES-2003-017). Washington, DC: National Center for Education Statistics.

Taylor, Janet A. & Mohr, Joan (2001). Mathematics for math anxious students studying at a distance. *Journal of Developmental Education, 25*(1), 30-37.

Testone (1999) On-line courses: a comparison of two vastly different experiences or ????. *Research and teaching in developmental education,* 16(1), 93-97.

Thompson (2004) Evaluating online courses and programs. *Journal of computing in Higher Education.* 15(2). 63-84.

Ungerleider, C. & Burns, T. (2003). *A systematic review of the effectiveness and efficiency of networked ICT in education: A state of the art report to the Council of Ministers Canada and Industry Canada.* Ottawa, Ontario, Canada: Industry Canada.

Winkelmann, C.L. (1995). Electronic literacy, critical pedagogy, and collaboration: a case for cyborg writing. *Computers and the Humanities, 29,* 431-448.

Weems, Gail H (2002). Comparison of beginning algebra taught onsite versus online. *Journal of Developmental Education, 26*(1), 10-18.